COMPENSATION OF SELECTIVE AVAILABILITY ERRORS OF GLOBAL POSITIONING SYSTEMS

by

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Certificate

It is certified that the work contained in the thesis entitled COMPENSATION OF SELECTIVE AVAILABILITY ERRORS OF GLOBAL POSITIONING SYSTEMS by *Sqn Ldr A.J.Devakumar* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Abstract

In satellite based Global Positioning System (GPS), satellite operators may intentionally degrade the navigational accuracy of civilian users of the system. This is is done by dithering the satellite clocks and manipulating the satellite ephemerides, and is called Selective Availability(SA).These errors may be largely eliminated by differential operation of the GPS. However DGPS is effective within limited range of reference stations. In the present study, a GPS scheme is proposed which features the use of multi-site reference GPS stations of precisely known locations, spread over the Indian sub-continent from which range measurements to the satellites are made. The equations relating the broadcast satellite positions and the pseudorange measurements to the SA errors are linearised and solved for SA errors. It is envisaged that these errors may be broadcast so that all user GPS receivers may compensate for errors and determine their positions with accuracy not possible under SA.

The procedure is illustrated through a simulated exercise involving the satellites of the NAVSTAR GPS at five epochs (instants of time) and eight ground reference stations, in which the SA errors are simulated by randomly chosen distances of departure from true satellite positions. It was found that under the assumptions of ignoring the errors in pseudorange measurements the true positions of the satellites may be determined to sub meter accuracy. However when measurement errors were introduced, the scheme was not successful. The estimated SA errors were very high.

The effect of SA and measurement errors on point positioning of user GPS receiver

was studied. It was found that both SA errors and measurement errors propagate to errors in positioning, without high amplification.

In the differential operation of GPS (DGPS) two direct methods of correction were studied. It was found that the position correction transfer method was more effective than the range correction transfer, in reducing the positional errors of user GPS receiver.

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List of Symbols

c	speed of light
e	eccentricity of orbit
E	eccentric anomaly of Keplerian orbit
La	latitude in degree
Lo	longitude in degree
M	mean anomaly of Keplerian orbit
n	mean angular velocity of satellite
R	radius of earth
R_e	estimated range
R_t	true range
t_a	reference epoch within current week
t_e	ephemerides reference epoch
t_o	beginning of the current GPS week
t_s	reading of the satellite clock at signal emission time
t_r	reading of the receiver clock at signal reception time
T_o	time of perigee passage
x_e,y_e,z_e	estimated positions in x,y,z coordinates
x_t,y_t,z_t	true positions in x,y,z coordinates
X_s,Y_s,Z_s	satellite coordinate in x,y,z axis
X_r,Y_r,Z_r	receiver coordinates in x,y,z axis

Δx,Δy,Δz	errors in x,y,z coordinates
δ_{sT}	total satellite clock error including error due to SA
δ_s	satellite clock error without SA
δ_{sA}	satellite clock error due to SA
Δt	difference between satellite and receiver clock readings
θ	true anomaly of Keplerian orbit
ω	argument of perigee
Ω	right ascension of the ascending node
ω_e	angular velocity of earth
ρ	pseudorange
δ	user-reference station separation
δ_i	offset from 0.3 semicircles($\approx 54^o$)

Subscripts

i	related to satellite
j	related to reference ground station receiver
k	related to user receiver

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Chapter 1

Introduction

Navigation is the art of directing the movement of a vehicle from one place to another on the earth or in space. Historically, there have been three independent types of navigation .

(a) Celestial- In this, present position is computed by measuring the elevation angles, or altitudes of stars and noting the time of observation. The interest in celestial navigation in aircraft is increasing due to the development of star trackers as effective during the day light as at the night and increasing use of airborne computers, which point the tracker and calculate the vehicle position. The disadvantage of the system is that the present daylight star trackers have narrow fields of view and examine only small portion of the sky at one time [10]. The "wide-angle" detector would examine large regions of the whole sky and identify a desired star by pattern recognition.

(b) Dead reckoning- Here course and distance travelled from the point of departure are maintained by plotting on a chart, or by continuous computation of north-south and east-west components from heading and speed of the vehicle. The Inertial Navigation System (INS) has emerged as a very useful extension of the dead-reckoning system. In INS the accelerations and attitudes/angular velocities of the vehicle are measured, and the displacements from initial station computed by integration. INS is self sufficient in the sense that it requires no external aids and it has high short term accuracy . However the position and velocity information degrades with time

and periodic alignment is necessary. Further-more INS is unstable in the vertical mode , i.e., determination of altitude, and requires supplementing by barometric altimeter. INS used in conjunction with other navigational systems is very effective.

(c) Piloting- This technique makes use of radio navigation, land -marks and visual patterns on the earth's surface. Radio navigation using electromagnetic waves was used since world war II. The commonly used radio navigation systems are Very high frequency Omnidirectional Ranging (VOR), Distance Measuring Equipment (DME), Tactical Air Navigation (TACAN) and Long Range Navigation (LORAN). These radio systems are intended for position fixing in one or two dimensions, during en-route navigation on earth, with the altitude of the aircraft small in comparison to its distance from the navigational facility. Need for line of sight visibility of a ground station from aircraft, for VOR,DME etc., and electromagnetic interference etc. limit the effectiveness of Radio Navigation systems. These however, presently are the most widely used air navigational aids.

1.1 Global Navigation Satellite System (GNSS)

A significant technological breakthrough occurred when scientists around the world experienced that the Doppler shift in the signal broadcast by a satellite could be used as an observable to determine the exact time of closest approach of the satellite. This knowledge, together with the ability to compute satellite ephemerides according to Kepler's laws, led to the capability of instantaneouly determining precise position anywhere in the world using satellites. The first operational satellite-based navigation system was called NNSS (Navy Navigation satellite System) or Transit [1]. Transit was based on a user measuring the Doppler shift of a continuous tone broadcast at about 400 MHz by polar orbiting satellites at altitudes of about 600 nautical miles. The maximum rate of change in the Doppler shift of the received signal corresponded to the point of closest approach of the Transit satellite. The difference between "up" Doppler and "down" Doppler can be used to calculate the range to the satellite at closest approach. The main problem with Transit system was the large time gaps in coverage. Since nominally a satellite would pass overhead every 90 minutes, users had to interpolate their position between "fixes"

or passes. The second problem with the Transit system was its relatively low navigation accuracy.

Presently, there are two Global navigation satellite syatems are available. They are

(a) NAVigation System Timing and Ranging Global Positioning System (NAVSTAR GPS)

(b) Global Navigation Satellite System (GLONASS)

NAVSTAR GPS was developed by US Department of Defence (DoD) and GLONASS by Russia. These systems are based on radio ranging to a constellation of artificial satellites. The system provides a continuous global positioning capability, with the help of 21 evenly spaced satellites placed in circular 12-hour orbits inclined 55^0 to the equatorial plane.

GLONASS offers many features in common with Navstar GPS. Its orbital plan forsees 24 satellites forming the space segment. It uses 3 orbital planes separated by 120^o of longitude and with equal spacing between satellites of 45^o within the plane. The orbits are near-circular with a period of around 11.25 hours at a height of 19100 km and an inclination of 64.8^o. In addition to these systems, the European Space Agency(ESA) is designing its own space-based radio navigation system, designated "ESA/NAVSAT". At present, the ESA/NAVSAT is only in study phase [4].

1.1.1 GPS: Principle of ranging and velocity determination

The NAVSTAR GPS satellites are configured, primarily, to provide the user with the capability of determining his position, expressed for example by latitude, longitude, and elevation by using the principle of ranging, and his velocity by using Doppler principle.

At a given instant, the space coordinates $\underline{\rho}_i$ of the satellites relative to the centre of the earth can be computed from ephemerides broadcast by the satellites. If the ground receiver defined by the geocentric position vector $\underline{\rho}_k$ employed a clock that was set precisely to GPS system time, the true distance or range R_t to each satellite could be accurately measured by recording the time required for the satellite signal to reach the receiver. Each range defines a sphere with the centre at the satellite. Hence, using this technique, ranges to only three satellites would be needed since the intersection of three spheres yields the

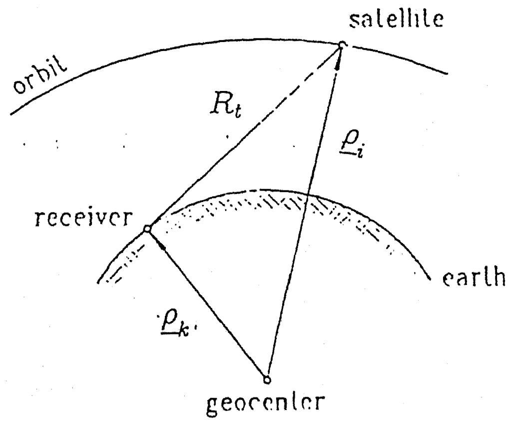


Figure 1.1: Principle of point positioning by ranging

three unknowns (e.g., latitude, longitude and height) which could be determined from the three range equations $R_t = \|\underline{\rho}_i - \underline{\rho}_k\|$. But, if the ground receiver clock is offset from the satellite clock/system clock, then the distance measured to the satellite is slightly longer or shorter than the "true" range. The receiver can overcome this problem by measuring the distances to four satellites(simultaneouly). These distances are called pseudoranges ρ since they are the true range plus a small(positive or negative) range correction ΔR_t resulting from the receiver clock error or bias δ. A simple model for the pseudorange(which is also called code pseudorange) is $\rho = R_t + \Delta R_t = R_t + c\delta$, where c being the velocity of light.

The pseudoranges can also be found out by measuring the phase of the incoming satellite signal. This range is called the phase pseudorange. The accuracy of the phase pseudorange is greater than the code pseudorange because the phase of satellite signal(carrier) can be measured to better than 0.01 cycles which corresponds to millimeter precision.

The determination of the instantaneous velocity of the receiver can be achieved using Doppler principle. Because of the relative motion of the satellites with respect to a moving vehicle, the frequency of a signal broadcast by the satellites is shifted when received at the vehicle. This measurable Doppler shift is proportional to the relative radial velocity. Since the radial velocity of the satellites is known, the radial velocity of the moving vehicle along the radial to each satellite can be deduced from the Doppler observable.

1.2 Recent developments in satellite navigation

Presently, NAVSTAR GPS satellite navigation system is widely used. All types of aircraft and vessels use GPS for enroute navigation. It has been demonstrated that preicse positioning in the centimetre range is possible while on the move or in other words 'on-the-fly'[7]. Thus it is possible, in the future, that GPS can be used as an aircraft landing aid. In the spring of 1994, Federal Aviation Administration (FAA) has officially accepted GPS as part of the US airspace system. In surveying GPS provides a powerful geodetic tool. It is used to measure crustal deformations, volcanic uplift and plate tectonics. Another use of GPS is in global determination of accurate time. High accuracy timing has many scientific applications such as coordinating seismic monitoring and other global geophysical measurements. Presently, the achievable accuracy of time transfer via GPS is one nanosecond. It is possible to integrate other navigational systems such as Inertial Navigation System with GPS in order to improve reliability and integrity .

1.3 Present investigation

United States Department of Defence (US DoD) can intentionally degrade the navigation accuracy of GPS system by dithering the satellite clock and manipulating the ephemerides data of satellites. This is called Selective Availability(SA). Due to SA the ranging error will increase and hence navigational accuracy will reduce. The present study consists of three parts: a) a scheme to determine the SA errors of satellites by measurement of pseudoranges from multi-site reference stations. b) effect of SA errors on the point positioning performance of GPS (stand-alone GPS). c) performance of GPS in differential mode. By generating simulated data of GPS satellites and reference stations the schemes and attendant procedures are illustrated. In all the cases, the effect of random errors present in the measurement of pseudoranges were simulated.

Chapter2 gives an overview of GPS system, chapter3 deals with the method to evaluate the SA errors and random error analysis. In chapter4 we present the results and their dicussion, and in chapter5 conclusions and recommendations for future work.

Chapter 2

Global Positioning System - an overview

2.1 Basic concepts of Navstar GPS

The Navstar Global Positioning System is an all-weather, space-based navigation system developed by United States Department of Defense(US DoD) to satisfy the requirements for the military forces to accurately determine their position, velocity, and time in a common reference system, anywhere on or near the earth on a continuous basis [3]. To provide a continuous global positioning capability, a scheme to orbit sufficient number of satellites to ensure that four were always electronically visible, was developed for GPS.

The GPS system consist of three segments.

(a) Space segment consisting of satellites which broadcast signals.

(b) Control segment steering the whole system which consists of master control station, worldwide monitor stations, and ground control stations.

(c) User segment which includes various types of receivers for military/civil users.

2.1.1 Space segment

Constellation

The space segment will provide global coverage with four to eight simultaneous observable satellites above 15^0 elevation. This is accomplished by satellites in nearly circular orbits with an altitude of about 20200 km above the earth and a period of approximately 12 sidereal hours [3]. The constellation consists of 21 operational satellites plus three active spares deployed in six planes with an inclination of 55^0 and with four satellites per plane.

GPS satellites

The GPS satellites essentially, provide a platform for radio transreceivers, atomic clocks, computers and various ancillary equipment used to operate the system. The electronic equipment of each satellite allows the user to measure a 'pseudorange' ρ to the satellite, and each satellite broadcasts a message which allows the user to determine the spatial position ρ_s of the satellite for arbitrary instants. Using these capabilities, users will be able to determine their position ρ_k on or above the earth.

There are five classes or types of GPS satellites. These are the Block I, Block II, Block IIA, Block IIR, and Block IIF satellites. Block I satellites were initially launched by US DoD, in the period between 1978 to 1985. The Block II constellation is slightly different from the Block I constellation since the inclination of their orbital planes is 55^0 compared to the former 63^0 inclination. Apart from orbital inclination, there is an essential difference between Block I and Block II satellites related to U.S. national security. Block I satellite signals were fully available to civilian users, while some Block II satellite are restricted. The Block II satellites were designated for the first constellation (i.e., 21 active and 3 spare active satellites). The Block IIA satellites ("A" denotes advanced) are equipped with mutual communication capability. Some of them can be tracked by Laser ranging. The GPS satellites which will replace the Block II's are the BlockIIR's. The "R" denotes replenishment or replacement. These satellites are currently under development. The next generation satellites, the Block IIF satellites ("F" denotes follow on), will be launched in the period between 2001 and 2010. These satellites will have improved on-board capabilities such as inertial navigation systems.

The Initial Operational Capability (IOC) was attained in December 8,1993 when 24

GPS satellites (Block I/II/IIA) were operating and were available for navigation. The US Air Force Space Command (AFSC) formally declared the GPS satellite constellation as having met the requirements for Full Operational Capability (FOC) as of April 27, 1995 [16]

Satellite signal

The oscillators on board the satellite generate a fundamental frequency f_o (10.23 MHz) with a stability in the range of 10^{-13} over one day for the Block II satellites. Two carrier signals in the L-band, denoted L1($154f_0$=1575 MHz) and L2($120f_0$=1227 MHz), are generated by integer multiplications of f_0 [1]. These signals are generated synchronously. Potentially, both the signal at the L1 frequency and signal at L2 can each have two modulations at the same time. Current implementation has two modulations on L1 frequency, but only a single modulation on L2 frequency. The two modulations are charaterised by a pseudorandom noise (PRN) sequence code.They are

(a) C/A or Clear/Coarse Acquisition code [1]- It is a short PRN code broadcast at a bit rate of 1.023 MHz and is repeated every millisecond. This is the principal civilian ranging signal, and it is always broadcast in the clear(unencrypted)form. The use of this signal is called the Standard Positioning Service. It is also used to acquire the much longer P code(described below).

(b) P or Precise code [1]- It is a very long code, (segments of a 200 day code) which is broadcast at ten times the rate of C/A, 10.23 MHz. Because of its higher modulation bandwidth, the P-code ranging signal is somewhat more precise. This reduces the noise in the received signal. This signal provides the Precise Positioning Service or PPS. The US Department of Defence has decided to encrypt this signal in such a way that it will not be available to the unauthorised user. When encrypted, the P code becomes the Y code.

In addition to the PRN codes a data message also called the navigation message, is modulated onto the carriers comprising satellite ephemerides, ionospheric modelling coefficients, status information, system time, satellite clock bias and drift information. This data message is transmitted at a frequency of 50 Hz and conveys necessary information

to lock on to the P code after acquiring the C/A code. In addition to this, the navigation message contains almanac data and broadcast ephemerides which includes various correction data.

Almanac data

The purpose of the almanac data is to provide the user with less precise data to facilitate receiver satellite search or for planning tasks such as the computation of visibility charts. The almanac data essentially contains parameters for the orbit representation, satellite clock correction parameters, and some other information as given in Table 2.1

The parameter ℓ_0 given in Table2.1 denotes the difference between the node's right ascension at epoch t_a and the Greenwich sideral time at t_0, the beginning of the current GPS week. The reduction of the Keplerian parameters to the observation epoch t is obtained by the formulas

$$\begin{aligned} M &= M_0 + n(t - t_a) \\ i &= 54^o + \delta i \\ \ell &= \ell_0 + \dot{\Omega}(t - t_a) - \omega_E(t - t_0) \end{aligned}$$

where ω_E is the angular velocity of earth. An estimate for the satellite clock bias is given by

$$\delta^S = a_0 + a_1(t - t_a)$$

Broadcast ephemerides

The broadcast ephemerides in Table 2.2 are based on observations at the five monitoring stations of the GPS control segment. The most recent of these data are used to compute a reference orbit for the satellites. Additional tracking data are entered into a Kalman filter and the improved orbits are used for extrapolation.

These orbital data could be accurate to approximately 5 m based on three uploads per day; with single daily update one might expect an accuracy of 10 m. The Master Control Station is responsible for the computation of the ephemerides and the upload to

Table 2.1: Alamnac data[3]

parameter	Explanation
ID	Satellite PRN number
HEALTH	Satellite health status
WEEK	Current GPS week
t_a	Reference epoch in seconds within the current week
$\sqrt{a}$	Square root of semimajor axis
e	Eccentricity
M_0	Mean anomaly at reference epoch
ω	Argument of perigee
δ_i	Offset from 0.3 semicircles($\approx$54^o)
ℓ_0	Longitude of node at weekly epoch
$\dot{\Omega}$	Rate of node's right ascension
a_0	Clock phase bias
a_1	Clock frequency bias

Table 2.2: Ephemeris data [3]

Parameter	Explanation
AODE	Age of ephemerides data
t_e	Ephemerides reference epoch
Δn	Mean motion difference
i	Rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
C_{uc},C_{us}	Correction coefficients (argument of perigee)
C_{rc},C_{rs}	Correction coefficients (geocentric distance)
C_{ic},Cis	Correction coefficients (inclination)
$\left.\begin{matrix}\sqrt{a}, & e, & M_0 \\ \omega_0, & i_0, & \ell_0\end{matrix}\right\}$	Keplerian parameters at t_e

the satellites. Essentially the ephemerides contain six parameters to describe a smoothed Kepler ellipse at a reference epoch and some secular and periodic correction terms. The most recent parameters listed in Table 2.2 are broadcast every hour and should only be used during the prescribed period of approximately four hours to which they refer.

The perturbation effects due to the nonsphericity of the earth, the direct tidal effect, and the solar radiation pressure are considered by the last nine terms in the Table2.2. In order to compute the satellite's position at the observation epoch t, the following quantities, apart from the parameters a and e, are needed

$$\begin{aligned}
M &= M_0 + \left[\sqrt{\frac{\mu}{a^3}} + \Delta n\right](t - t_e) \\
\ell &= \ell_0 + \dot{\Omega}(t - t_e) - \omega_E(t - t_0) \\
\omega &= \omega_0 + C_{uc}cos(2u) + C_{us}sin(2u) \\
r &= r_0 + C_{rc}cos(2u) + C_{rs}sin(2u) \\
i &= i_0 + C_{ic}cos(2u) + C_{is}sin(2u) + i(t - t_e)
\end{aligned}$$

where u= $\omega + \theta$ is the argument of latitude. The r_0 is the geocentric distance calculated using a, e, E at the observation epoch.

2.1.2 Control segment

The main operational tasks of the control segment are tracking of the satellites for the orbit and clock determination and prediction modelling, time synchronisation of the satellites, and upload of the data message to the satellites. This segment comprises of master control station, worldwide monitor stations, and ground control stations.

The Master Control Station collects the tracking data from the monitor stations and calculates the satellite orbit and clock parameters. These results are then passed to one of the three ground control stations for eventual upload to the satellites. The satellite control and system operation is also the responsibility of the master control station.

There are five [3] Monitoring Stations. Each of these stations is equipped with a precise cesium time standard and P-code receivers which continuouly measure the P-code pseudoranges to all satellites in view. Pseudoranges are tracked every 1.5 seconds and using the ionospheric and meteorological data, they are smoothed to produce 15 minute

interval data which are transmitted to the master control station.

The Ground Control Stations are collocated with the monitor stations and they are the communication links to the satellites. They mainly consist of the ground antennas. The satellite ephemerides and clock information, calculated at the master control station and received via communication links, are uploaded by the ground control stations to each GPS satellite via S-band radio links [3].

2.1.3 User segment

The user segment consists of GPS receiver units. The receivers process the signal which are transmitted by GPS satellites. The receiver unit contains elements for signal reception and signal processing. Based on the type of observables(i.e., code pseudoranges or carrier phases) and on the availability of codes(i.e., C/A code or P code) , GPS receivers are classified into three groups.

(a) C/A code pseudorange

(b) C/A code carrier phase

(c) P code carrier phase

C/A code pseudorange receivers

This type of receiver is usually a hand held device powered by flashlight batteries [3]. Typical devices have from one to six independent receiver channels and output the three-dimensional position(i.e., longitude, latitude and height). Receivers with four or more channels are preferred for applications where the receiver is in motion since simultaneous satellite ranges can be measured to produce more accurate positions. On the other hand, a single channel receiver is adequate for applications where the receiver is at a fixed location and the range measurements can be sequentially determined.

C/A code carrier phase receivers

These receivers measure the phases of the L2 carrier by the use of the codeless squaring technique. This is accomplished by multiplying the L2 signal by itself to recover the phase

of the carrier at one-half its wavelength. The P-code modulated on the L2 carrier is lost in the process and the signal to noise ratio is considerably lower than the C/A-code L1 measurement.

P-code receivers

This type of receiver uses the P-code and, thus, enables lock on to the L1 and L2 carrier wave. The P-code data on the carrier waves are derived by correlating the signals with a replica of the P-code generated in the receivers. To do this, the structure of the code must be known. The cross correlation is performed by matching the received satellite signal with the receiver generated code replica. No carrier phase measurement can be performed before the code components of the received satellite signal are removed [3].

2.1.4 Processing techniques

The aim of the various processing techniques are to deduce code pseudorange or phase pseudorange based on a comparison between received signals and receiver generated signals. In phase pseudorange measurements when a receiver is turned on, the fractional beat phase (i.e., the phase difference between the satellite transmitted carrier and a receiver generated replica signal) is observed. The initial integer number N of cycles between satellite and receiver is unknown. However, when tracking is continued without loss of lock, the number N, also called integer ambiguity, remains the same. Hence there are two problems with phase pseudorange measurement. Firstly, the integer ambiguity to be resolved. Secondly, in the case of loss of the signal lock the integer counter is reinitialized which causes a jump in the instantaneous accumulated phase by an integer number of cycles. This jump is called cycle slip which, of course, is restricted to phase measurements. Hence in order to make correct phase measurement cycle slips to be detected and repaired.

The actual pseudorange measurements are performed in tracking loop circuits. The code pseudoranges are determined in the Delay Lock Loop (DLL) by using the code correlation technique which is explained below. After removing the PRN code from the incoming signal and some filtering, the unmodulated (Doppler shifted) carrier wave is obtained. This carrier wave is then passed to the Phase Lock Loop(PLL) where the phase

measurement is conducted. The result is the (fractional) phase offset between the received signal and the reference signal generated in the receiver.

The code correlation technique provides all components of the satellite signal: the satellite clock reading, the navigation massage, and the unmodulated carrier. The code correlation requires the knowledge of one PRN code. This is performed in several steps. First, a reference carrier is generated in the receiver which then is biphase modulated with a replica of the known PRN code. In a second step, the resulting reference signal is correlated with the received satellite signal. The signals are shifted with respect to time so that they optimally match (based on a mathematical correlation). The necessary time shift Δt, neglecting clock biases, corresponds to the signal travel time from the satellite antenna to the phase centre of the receiving antenna. After removing the PRN code, the received signal still contains the navigation message which can be decoded and eliminated by high-pass filtering. The final result is the Dopplar shifted carrier on which a phase measurement can be performed. Since a PRN code is required, the code correlation technique is generally only applicable to the C/A-code with only the L1 carrier being reconstructed. One result of the C/A-code correlation is the decoded navigation message which contains the Hand over Word (HOW) in each subframe which tells the receiver where to start the search in the P-code for signal matching.

Without knowledge of the P-code (more precisely, the Y-code), one has to apply codeless techniques for the reconstruction of the unmodulated carrier wave from which the phase of the base carrier is measured. Most of the receivers provide a hybrid technique. The L1 carrier is reconstructed by code correlation using the C/A-code, and a codeless technique is applied to reconstruct the L2 carrier.

2.2 Performance of GPS

The performance capabilities of GPS are primarily affected by three factors:

(a) Denial of accuracy and access

(b) Ranging errors

(c) satellite geometry (which causes "geometric dilution")

2.2.1 Denial of accuracy and access

There are basically two methods for denying civilian users full use of the system. These are used to degrade the accuracy of the GPS for civilian users due to security reasons. The first is Selective Availability (SA) and the second method is Anti-spoofing (A-S).

Selective Availability

The accuracy expected from C/A-code pseudorange positioning at present is 15-40m [3] and a fraction of a meter per second for velocity. The goal of SA is to degrade this accuracy by dithering the satellite clock (δ-process) and manipulating the ephemerides (ε-process). Following the specifications of the DoD, the accuracy is degraded to 100m for horizontal position and to 156m for height. These specifications also imply a velocity error of 0.3 meter per second and an error in time of 340 ns [3]. All numbers are given at the 95% probability level. At the 99.99% probability level, the predictable accuracy decreases to 300m for horizontal position and to 500m for height.

The δ-process is achieved by introducing varying errors into the fundamental frequency of the satellite clock. The satellite clock bias has a direct impact on the pseudorange, which is derived from a comparison of the satellite clock and receiver clock. Since the fundamental frequency is dithered, code and carrier pseudoranges are affected in the same way. In fig2.1, the different behaviour of satellite clocks with and without SA is shown. With activated SA, there are variations of the pseudoranges with amplitudes of some 50m and with periods of some minutes.

The ε-process is the truncation of the orbital information in the transmitted navigation message to that the coordinates of the satellites cannot accurately be computed. The error in satellite position roughly translates to a like position error of the receiver. The orbital errors cause pseudorange errors. In fig2.2, the behaviour of the radial orbit error with and without SA is shown. In the case of SA, there are variations of the pseudoranges with amplitudes between 50m and 150m and with periods of some hours.

Anti-Spoofing

The design of GPS includes the ability to essentially "turn off" the P-code or invoke an encrypted code as a means of denying access to the P-code to all but authorized users.

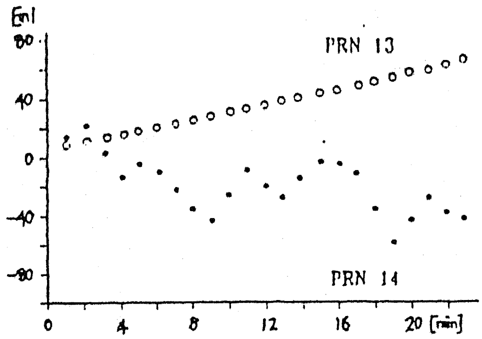


Figure 2.1: Satellite clock behaviour of PRN 13(without SA) and of PRN 14(with SA)

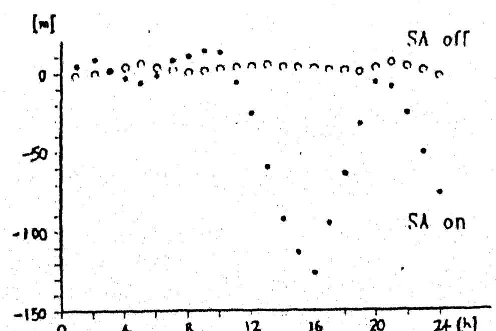


Figure 2.2: Radial orbit error of PRN 21 with and without SA

The rationale for doing this is to keep adversaries from sending out false signals with the GPS signature to create confusion and cause users to misposition themselves. This is called Anti-Spoofing (A-S).

A-S is accomplished by the modulo 2 sum of the P-code and an encrypting W-code. The resulting code is denoted as Y-code. Thus, when A-S is active, the P-code on the L1 and the L2 carrier is replaced by the unknown Y-code. A variable influence of A-S (as is the case with SA) cannot occur.

2.2.2 Ranging errors

The ranging errors, which occur during the measurement of range from satellite to the receiver, are grouped into six major causes. They are

(a) Satellite Ephemeris

(b) Satellite clock

(c) Ionospheric Group Delay

(d) Tropospheric Group Delay

(e) Multipath

(f) Receiver Measurement Errors-including software.

In ranging error, some are biases, some slowly varying and some rapidly varying. Again some of these errors tend to be correlated for the same satellite. For example, satellite clock and ephemeris errors tend to be negatively correlated, i.e. they tend to cancel each other somewhat [1]. Other errors tend to be correlated between satellites (e.g. ionospheric and tropospheric delays).

The ranging error due to satellite ephemeris error will be of the order of 4.0 m [5] and of satellite clock error will be 1.5 m [5].

Ionospheric group delay

The ionosphere, extending in various layers from about 50 km to 1000 km above earth, is a dispersive medium with respect to the GPS radio signal. Due to this the GPS code

measurements are delayed and the carrier phases are advanced. Therefore, the code pseudoranges are measured too long and the carrier phase pseudoranges are measured too short compared to the geometric range between the satellite and the receiver. The amount of the difference is in both cases the same. The ionospheric group delay can be eliminated by two methods. They are

(a) By modelling the atmosphere

(b) Using two signals with different frequencies. This dual frequency method is the main reason for the GPS signal has two carrier waves L1 and L2.

Tropospheric group delay

The effect of the neutral atmosphere (i.e. the nonionized part) is denoted by tropospheric group delay. The neutral atmosphere is a nondispersive medium with respect to radio waves up to frequencies of 15 GHz, and thus, the propagation is frequency independent. The disadvantage is that an elimination of the tropospheric refraction by dual frequency method is not possible. Tropospheric delay can be compensated by using suitable atmospheric models.

Multipath errors

The multipath errors occur due to fact that a satellite signal arrives at the receiver via more than one path. Multipath is mainly caused by reflecting surfaces near the receiver. The P-code receivers reject reflected signals whose path delay exceeds 150 feet. For the C/A signal, the number is 1500 [1] feet. The multipath errors are frequency dependent and the phase differences are proportional to the differences of the path length. The elimination of multipath signals can be done by various methods.

(a) By avoiding, as far as possible, reflecting surfaces in the neighbourhood of the receivers.

(b) By selecting a receiver antenna that takes advantage of the signal polarisation. GPS signals are right-handed circularly polarized, whereas the reflected signals are left-handed polarized.

(c) By digital filtering, using wideband antennas and radio frequency absorbent antenna ground plane.

Receiver measurement errors

This includes Antenna phase center variation and clock bias. The phase center of the antennas is the point to which the radio signal measurement is referred and generally is not identical with the geometric antenna center. The offset depends on the elevation, the azimuth, and the intensity of the satellite signal and is different for L1 and L2. The precision of an antenna should be based on the antenna phase center variation. A constant offset could easily be determined and taken into account.

Thus the code pseudoranges and phase pseudoranges are affected by both, systematic errors or biases and random noise. Note that Doppler measurements are affected by the rate of change of the biases only. The error sources can be classified into three groups, namely satellite related errors, propagation related errors, and receiver related errors. The random noise mainly contains the actual observation noise plus random constituents of multipath. The measurement noise, an estimation of the satellite biases, and the contributions from the wave propagation are combined in the User Equivalent Range Error (UERE). This UERE is transmitted via the navigation message.

2.2.3 Dilution of Precision

Another factor which affects the performance of GPS is the satellite geometry. The geometry of the visible satellites is an important factor in achieving high quality results. The geometry changes with time due to the relative motion of the satellites. A measure for the geometry is the Dilution of Precision(DOP) factor. The variations in dilution factor are typically much greater than the variations in ranging errors. The DOP factor is inversely proportional to the volume of a body. This body is formed by the intersection points of the site-satellite vectors with the unit sphere centered at the observing site. The larger the volume of this body, the better the satellite geometry. Since good geometry should mirror a low DOP value, the reciprocal value of the volume of the geometric body is directly proportional to DOP.

An ideal geometry would be three low-orbiting satellites 120 degrees apart in azimuth,

with the fourth at the zenith. Infact, positional error is approximately equal to the appropriate DOP value times the pseudorange error. If horizontal positioning is of interest, then Horizontal DOP (or HDOP) should be used. If application requires three-dimensional positioning, then Positional DOP (or PDOP) should be used as the proportionality constant.

2.3 Differential GPS

2.3.1 Principle of operation

As we have seen, most of the ranging errors in Standard Positioning Service are slowly varying. This feature make it possible to greatly enhance the accuracy of GPS in local areas through Differential GPS (DGPS). A DGPS system employs a local reference station, which has a high-quality GPS receiver and an antenna at a known, surveyed location. i.e. the accurate position of the receiver is known. The reference station estimates the slowly varying components of the satellite range measurement errors, and transmits them as corrections to users within communications range of the station. Users near the station are expected to achieve accuracies of 2 to 8 meters [5], depending on platform dynamics and receiver/processor sophistication. The accuracy of the corrections is reduced as the user-reference station separation increases (spatial decorrelation) and time passes (temporal decorrelation). Additionally, the reference station in DGPS will act as a monitor, and can inform users of problems that might arise in the use of the satellite signals. Incidentally, while the achievable accuracies are better than the Precise Positioning Service, the DGPS does not pose a security problem because the corrections are available and accurate only over a limited area.

2.3.2 Ranging errors in differential GPS

As stated earlier, the ranging errors have many sources; some are biases, some slowly varying, and some rapidly varying. Differential GPS operation can remove most of the bias and slowly varying errors, so that the ultimate accuracy is limited by the rapidly varying errors. In differential operation, the reference station computes the pseudorange to each satellite as usual. However, it also computes the true range to each satellite (with

S/A off), based on knowledge of the satellites' locations and the location of the station GPS antenna phase center. The difference between the measured pseudorange and the true range is the error (within a common bias) associated with each satellite. These errors are broadcast as corrections to the users of differential GPS.

Now we will examine, the effect of various sources of ranging errors, on the differential operation of GPS.

Selective Availability errors

As we have seen the SA can involve a combination of signal "dithering" and ephemeris data manupulation. In the first case, SA introduces slowly varying (but unknown) delay in to the time of signal transmission. If signal dithering alone is used, then the pseudorange error would be about 30 meters [5] (1σ). Additionally, the signal dithering would cause a differential correction to suffer from temporal decorrelation, which would amount to about 2 meters [5] of pseudorange error after about 30 seconds. Thus to keep positional error small (under 5 meters), the differtial corrections should be updated several times per minute.

In the second case, SA introduces a bias error into the satellite ephemeris data. This results in no temporal decorrelation, but spatial decorrelation does result. As shown in fig 2.3, if a 16 meter error in ephemeris is introduced by SA and the user is 1000 km from the reference station, then the differential correction would have an error of 0.8 meter. δ is the user-reference station separation.

Ionospheric group delay

The ionosphere can delay the satellite signal by as much as 30 meters under the following worst-case conditions: solar maximum and solar storms, at low elevation angles, and in the afternoon. In fact, a delay of 30 meters corresponds to a total electron content of 1.85×10^{18} electrons/m^2 at GPS L1 frequency. More typically, delays are in the 4 to 10 meter range. Even without differential corrections, about 50 to 75% of this error can be removed by using a standard model and coefficients available from the satellite data. With differential correction, the size of the residual pseudorange error depends on the separation of the user and the reference station. If a user is next to the reference station,

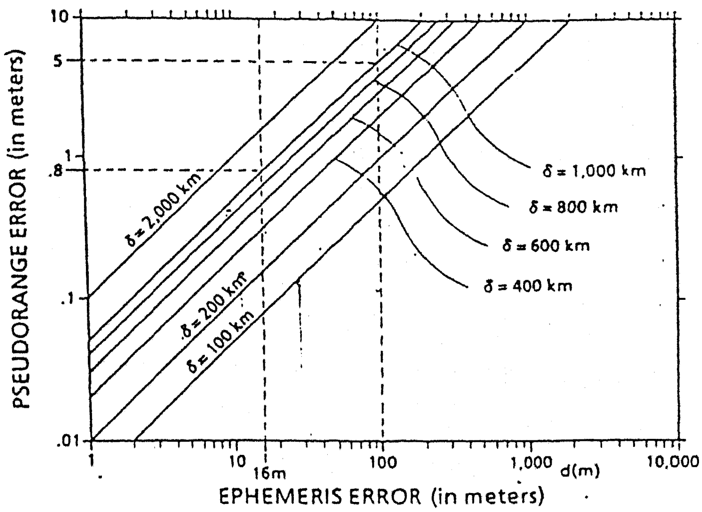


Figure 2.3: Pseudorange error introduced by satellite ephemeris error [5]

then the ionospheric delays will be identical, and differential GPS will remove their effect completely. However, at user-reference station separations of hundreds of kilometers, spatial decorrelation errors will probably be several meters [5]. The extent of ionospheric spatial decorrelation is not yet well known, and is a subject under active investigation.

Tropospheric group delay

The index of refraction of the lower atmosphere is not quite unity (typically 1.0003), and thus can result in a delay at very low satellite elevation angles (i.e., below about 10 degrees). A simple model predicts most of this effect [5], but unmodelled error can reach 2 to 3 meters at about 5 degrees elevation. While the signal ray paths to user and station can traverse quite different terrain, the differential tropospheric error is usually less than a meter, and varies very slowly. There is a small random component as well, but it is small.

User Receiver

Most GPS receivers use separate channels for each satellite, and each channel has a delay lock loop for estimating the corresponding pseudorange. Inter-channel bias errors also exist. The random errors are caused by background noise, thermal noise and multiple

access interference. Their size depends on signal-to-noise ratio, signal-to-interference ratio, tracking loop bandwidth and user acceleration . If the delay lock loop bandwidth is 1 Hz and the user moves slowly, then the standard deviation of these pseudorange errors can be upperbounded by 1.5 meters [5]. These errors are usually reduced by a "navigation" filter, which acts later in the signal processing. This filter reduces the magnitude of the random errors by a factor of 2 or so. Unfortunately, the user receiver errors cannot be further reduced by differential techniques because the errors are not correlated from receiver to receiver, and they have very small correlation times and distances.

Reference Receiver

Errors introduced by the GPS receiver at the reference station will be passed on to the DGPS service user. Such errors are the random errors and errors in the algorithms which generate the corrections. Once again, the standard deviation of these errors can be upper bounded by 1.5 meters, and further smoothing by a navigation filter can be reduce them by a factor of 2.

Multipath errors

Reflections from the earth's surface and nearby objects can cause errors in the user's position which DGPS would not eliminate. However, multipath errors are expected to be less than 1 meter (1σ) most of the time [5]. Since these errors have very small decorrelation distances, the differential GPS will not remove them.

comparison of ranging errors with and without Differtial corrections

The table2.3 summarizes the errors that contribute to both stand-alone GPS and differential GPS. In the table, it assumes the SA is used soley to dither time of transmission and is not applied to ephemeris data. Here δ is the user-reference station separation (in meters) and t is the age of the correction (in seconds). The total pseudorange error can be found by taking the square root of the sum of the individual errors squared. The total error (1σ) for a GPS user without differential corrections is approximately 30.8 meters. This means that selective availability is the dominant error source for the user without differential corrections

Table 2.3: Ranging errors with and without Differential corrections [5]

	Bias Errors wo/DGPS (meters)	Random Errors wo/DGPS (1σ,meters)	Bias Errors w/DGPS (meters)	Random Errors w/DGPS (1σ,meters)
Ephemeris Data	4.0	0.0	$2*10^{-7}\delta$	0.0
Satellite Clock Data	1.5	0.7	0.0	0.7
Ionosphere	4.0	0.0	$2*10^{-6}\delta$	0.0
Troposphere	0.0	0.5	0.5	0.5
Multipath	0.0	1.0	0.0	1.0
User Receiver	0.0	1.5	0.0	1.5
Selective Availability	30.0	0.0	$1.22*10^{-3}t^{2.12}; t \leq 40s$	0.0
Reference Receiver	0.0	0.0	0.0	1.5

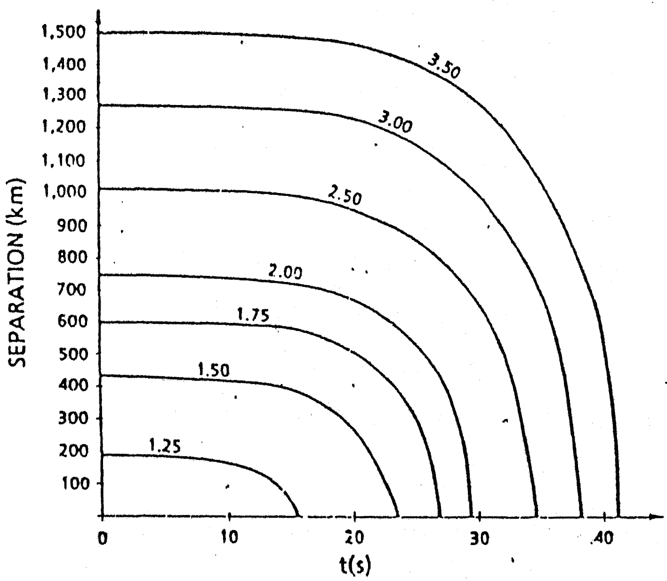


Figure 2.4: DGPS pseudorange errors versus age of correction and user/reference separation (meters)[5]

Fig. 2.4 shows the pseudorange error (1σ) for a user with differential corrections. The fig 2.4 presents equi-error contours as a function of user-reference station separation and the age of the correction. Fig 2.4 assumes that the random errors in fig 2.1 are smoothed by a navigation filter, which follows the delay lock loop and reduces the errors by a factor of 2.

2.3.3 Method of transmission of corrections in DGPS

The corrections consist of pseudorange and range rate terms. Latitude/longitude corrections are not broadcast because GPS receivers differ in how they select the satellite pseudoranges to be measured and how many satellites will be used. Consequently, a single latitude/longitude corrections would not serve the wide variety of receivers which will be in the field. The range rate corrections provides the user with a prediction of how the slowly varying SA errors are changing, thus improving the accuracy of DGPS. The pseudorange correction at the remote receiver for an epoch t may be calculated by

$$\Delta R(t) = \Delta R(t_0) + \frac{d\Delta R}{dt}(t - t_0)$$

where t_0 is the epoch of the data used for the computation of the corrections Δ R(t_0) at the reference station and (t-t_0) may be regarded as the age of the corrections.

The corrections will not use ionospheric or tropospheric models, which means that the user receiver must model the ray paths for both the user and reference station or neither. The differential message also includes the following information for each satellite: satellite identification, satellite health and estimated accuracy, and the "age" of the data being used by the reference station. This latter data element is included because the satellite data changes about once an hour, and different receivers take different lengths of time to incorporate the new data after such a change.

The auxiliary messages, some of which are required by all users, and others are designed for special applications. For example, the message which provides the users with the location of the DGPS station and carrier phase information, which is useful for surveying applications.

Chapter 3

Estimation of Selective Availability errors

As explained earlier the performance of the GPS is affected by Selective Availability. If SA errors can be determined, then the accuracy of the GPS receiver can be improved. The satellite position is estimated on the basis of the ephemeris data available in the navigation message. Then the distance from a surveyed, fixed location of a ground reference station to the estimated satellite position gives the estimated range to the satellite. Due to SA the true position of the satellite will differ from the estimated position of the satellite. Consequently the true range to the satellite will differ from the estimated range to the satellite by an amount which depends on the difference between the true potision and estimated position. The true range from a ground reference station to the satellite can be deduced from measured time or phase differences based on a comparison between received signals and receiver generated signals. The GPS uses two clocks, namely one in the satellite and other in the receiver. The range measurements are biased by satellite and receiver clock errors. The satellite clock error consists of satellite clock bias transmitted in the navigation message and clock error due to SA which is an unknown quantity. Let us denote by t_s the reading of the satellite clock at emission time and by t_r the reading of the receiver clock at signal reception time. The delays of the satellite and receiver clocks with respect to GPS system time will be termed δ_{sT} and δ_r respectively. The satellite clock

reading t_s is transmitted via the PRN code. The difference between the clock readings is equivalent to the time shift Δt which aligns the satellite and reference signal during the code correlation procedure in the receiver. Hence the true range from the ground reference receiver and the satellite is equal to the difference between the clock readings Δt multiplied by the speed of light c.

$\Delta t = t_r - t_s = [t_r(\text{GPS}) - \delta_r] - [t_s(\text{GPS}) - \delta_{sT}]$

where t_r(GPS) and t_s(GPS) are t_r and t_s referred to readings on GPS system clock.

$\Delta t = \Delta t(\text{GPS}) + \Delta\delta$

where $\Delta t(\text{GPS}) = t_r(\text{GPS}) - t_s(\text{GPS})$ and $\Delta\delta = \delta_{sT} - \delta_r$.

Hence the true range $R_t = c\Delta t(\text{GPS})$.

Here $\delta_{sT} = \delta_s + \delta_{sA}$

where δ_s is the satellite clock bias transmitted in the navigation message and δ_{sA} is the satellite clock bias due to SA.

3.1 Determination of true position of satellite

Let x_{ei}, y_{ei}, z_{ei} be the components of the estimated position vector of the ith satellite for epoch t, and let x_j, y_j, z_j be the coordinates of the jth ground reference station. The true satellite positions are given by

$$
\begin{aligned}
x_{ti} &= x_{ei} - \Delta x_i \\
y_{ti} &= y_{ei} - \Delta y_i \\
z_{ti} &= z_{ei} - \Delta z_i
\end{aligned}
$$

where Δx_i, Δy_i, Δz_i are the satellite position errors due to SA. The true range from ground reference station to the true position of the satellite can be represented by

$$
\begin{aligned}
R_{tij} &= \sqrt{(x_{ti} - x_j)^2 + (y_{ti} - y_j)^2 + (z_{ti} - z_j)^2} \qquad (3.1) \\
&\equiv f(x_{ti}, y_{ti}, z_{ti}) \\
f(x_{ti}, y_{ti}, z_{ti}) &\equiv f(x_{ei} - \Delta x_i, y_{ei} - \Delta y_i, z_{ei} - \Delta z_i)
\end{aligned}
$$

This can be expanded into Taylor series with respect to the estimated point.

$$R_{ti} = f(x_{ei}, y_{ei}, z_{ei}) \begin{aligned}[t] & - \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial x_{ei}} \Delta x_i \\ & - \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial y_{ei}} \Delta y_i \\ & - \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial z_{ei}} \Delta z_i \\ & - \cdots \end{aligned} \tag{3.2}$$

where the expansion is truncated after the linear term. The estimated rangeR_e is given by

$$\begin{aligned} R_{eij} &= \sqrt{(x_{ei} - x_j)^2 + (y_{ei} - y_j)^2 + (z_{ei} - z_j)^2} \\ &\equiv f(x_{ei}, y_{ei}, z_{ei}) \end{aligned}$$

The partial differentials are obtained by

$$\begin{aligned} \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial x_{ei}} &= \frac{x_{ei} - x_j}{R_{eij}} \\ \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial y_{ei}} &= \frac{y_{ei} - y_j}{R_{eij}} \\ \frac{\partial f(x_{ei}, y_{ei}, z_{ei})}{\partial z_{ei}} &= \frac{z_{ei} - z_j}{R_{eij}} \end{aligned}$$

Therefore the true range to the satellite is given by the linearised approximate equation

$$R_{tij} = R_{eij} - \frac{x_{ei} - x_j}{R_{eij}} \Delta x_i - \frac{y_{ei} - y_j}{R_{eij}} \Delta y_i - \frac{z_{ei} - z_j}{R_{eij}} \Delta z_i$$

$$R_{eij} - R_{tij} = \frac{x_{ei} - x_j}{R_{eij}} \Delta x_i + \frac{y_{ei} - y_j}{R_{eij}} \Delta y_i + \frac{z_{ei} - z_j}{R_{eij}} \Delta z_i$$

But the true range is also given by

$$R_{tij} = c\Delta t - c\delta s_i - c\delta_{sAi} + c\delta r_j$$

Here δ_{sAi} and δ_{rj} are unknowns. Therefore

$$\frac{x_{ei} - x_j}{R_{eij}} \Delta x_i + \frac{y_{ei} - y_j}{R_{eij}} \Delta y_i + \frac{z_{ei} - z_j}{R_{eij}} \Delta z_i - c\delta_{sAi} + c\delta_{rj} = R_{eij} - c\Delta t_{ij} + c\delta_{si}. \tag{3.3}$$

For each satellite there are 4 unknowns, namely Δx_i, Δy_i, Δz_i, δ_{sAi} and for each ground reference station there is one unknown i.e. δ_{rj}. Hence for I satellite and J ground reference station, there are IJ equations and 4I+J unknowns.

The set of linear equations can be written in the matrix-vector form

$\underline{A}\,\underline{x} = \underline{b}$

where $\underline{A}$ is IJ x (4I+J) matrix, $\underline{x}$ is 4I+J x 1 matrix and $\underline{b}$ is IJ x 1 matrix

$$\underline{A} = \begin{bmatrix} A_1 & \cdots & A_j & \cdots & A_J \end{bmatrix}^T$$

where

$$A_j = \begin{bmatrix} A_{j1} & A_{j2} & A_{j3} \end{bmatrix}$$

A_{j1} is I x 3I, A_{j2} is I x I, A_{j3} is I x J

$$A_{j1} = \begin{bmatrix} \alpha_{1j} & \beta_{1j} & \gamma_{1j} & 0 & . & . & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & \alpha_{2j} & \beta_{2j} & \gamma_{2j} & 0 & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & 0 & \alpha_{ij} & \beta_{ij} & \gamma_{ij} & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . & . & . & 0 & \alpha_{Ij} & \beta_{Ij} & \gamma_{Ij} \end{bmatrix}$$

$$A_{j2} = \begin{bmatrix} c & 0 & . & . & 0 \\ 0 & c & 0 & . & 0 \\ . & . & . & . & . \\ 0 & . & . & 0 & c \end{bmatrix}$$

$$A_{J3} = \begin{bmatrix} 0 & . & 0 & c & 0 & . & 0 \\ . & . & 0 & c & 0 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & c & 0 & . & 0 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$$

$$x_1 = \begin{bmatrix} x_1 1 & x_1 2 & . & x_1 i & . & x_1 I \end{bmatrix}^T$$

$$x_1 i = \begin{bmatrix} \Delta x_i & \Delta y_i & \Delta z_i \end{bmatrix}^T$$

$$x_2 = \begin{bmatrix} \delta_{sA1} & . & \delta_{sAi} & . & \delta_{sAI} \end{bmatrix}^T$$

$$x_3 = \begin{bmatrix} \delta_{r1} & . & \delta_{rj} & . & \delta_{rJ} \end{bmatrix}^T$$

$$\underline{b} = \begin{bmatrix} b_1 & b_2 & . & b_j & . & b_J \end{bmatrix}^T$$

$$b_j = \begin{bmatrix} b_{j1} & . & b_{ji} & . & b_{jI} \end{bmatrix}^T$$

$$b_{ji} = R_{e_{ij}} - c\Delta t_{ij} + c\delta_{Si}$$

Recalling that our objective is to determine the true positions x_{ti} etc. of each satellite i, and that $x_{ti} = x_{ei}$ - Δx_i etc., x_{ei} being the estimated position (along x coordinate) of the ith satellite obtained from the ephemerides broadcast by the satellite, and Δx_i etc. being the corresponding errors due to SA. We slove for Δx,Δy,Δz,δ_{sA} for each satellite and δ_r for each ground station from the above IJ equations. It may be seen by inspection, that in order to solve for the 4I+J unknown variables, from IJ equations, it is necessary that IJ $\geq$ 4I+J. Noting that for the determination of the position of a ground station atleast 4 satellites are required , the minimum number of satellites and ground stations required are I $\geq$ 4 and J $\geq$ 6. The solutions for the 4I+J unknowns may be obtained by judiciously choosing 4I+J equations, or by solving for these variables using all IJ equations to obtain the best estimate in some sense e.g., least square fit, of these variables. It may be noted that the IJ equations being linear approximations of the exact equations for $R_{eij} - R_{tij}$, are not exact, and therefore may not be strictly compatible with each other. We may therefore hope to obtain the best estimate of the variables compatible with the equations.

The accuracy of the solutions may however be improved by iteration. After making the first estimate of the errors Δx_i etc. one may correct the estimated values of x_{ei},R_{eij} hence update the appropriate coefficients of the variables which should be closer to the true values, if the computational process converges. Thus the errors in the positions and clocks of each satellite and the clock errors of each ground stations may be determined to the required degree of accuracy. The errors in position of each satellite thus determined, and the estimated positions of the satellites already given, the true positions of the satellite may be determined.

There are single-difference, double difference and triple difference methods to form linear combinations of equations between satellites and ground stations and thus unknowns can be solved. In single differnce, two ground stations and one satellite are required to form one equation. Double difference are obtained by forming equations between two ground sations and two satellites. In triple difference two ground stations and two satel-

lites at two epochs are necessary to form one equation.

3.2 Determination of user position

The aim of the scheme suggested above was to know the true position of the satellites so that ranging from satellite to the user can lead to improved accuracy in the user's position. In order to find the position of the user and user clock error there should be a minimum of 4 satellites visible from the user position. If an approximate/estimated position of the user receiver is known a linearised equation for the differences between the estimated range and the true range between the user receiver and the satellite in terms of errors in the position of the user receiver may be obtained on similar lines as those derived for errors in satellite positions at equation 3.3 . These may be written as

$$\frac{x_i - x_k}{R_{eik}}\Delta x_k + \frac{y_i - y_k}{R_{eik}}\Delta y_k + \frac{z_i - z_k}{R_{eik}}\Delta z_k + c\delta r = R_{eik} - c\Delta t - c\delta s_i \tag{3.4}$$

where

R_{eik} = range between the true position of ith satellite and approximate/estimated position of the user (k) .

Δt = time taken by the signal to travel between the true position of the satellite to the true position of the user.

δs_i = ith satellite clock error transmitted in the navigation message.

x_i,y_i,z_i = coordinates of ith satellite(true position).

x_k,y_k,z_k = coordinates of user (estimated)

$\Delta x_k,\Delta y_k,\Delta z_k$ = difference between the true position and estimated position of the user in the respective coordinates.

δr = the user clock error.

The quantities on the right-hand side are known: they are the differences between the estimated range, the time taken by the satellite signal to reach the receiver and satellite

clock bias transmitted in the navigation message. These equations may be expressed in matrix notation as

$$\begin{bmatrix} \alpha_{11} & \beta_{12} & \gamma_{13} & 1 \\ \alpha_{21} & \beta_{22} & \gamma_{23} & 1 \\ \alpha_{31} & \beta_{32} & \gamma_{33} & 1 \\ \alpha_{41} & \beta_{42} & \gamma_{43} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta y_k \\ \Delta z_k \\ c\delta r \end{bmatrix} = \begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ \Delta R_4 \end{bmatrix}$$

where α_{ik},β_{ik}, γ_{ik} are the direction cosine of the angle between the Line-of-Sight from the user receiver position to the ith satellite and the kth coordinate. This equation may be written more compactly as

$Ax = b$

where

A= the 4x4 matrix (i.e., a matrix of coefficients of the linear equation)

$x =$ user position and clock error vector

$b =$ the four element pseudorange measurement difference vector

There-fore $x = A^{-1}\ b$

Hence the true position of the user can be estimated.

3.3 Bounds of ranging error from the user

The satellite position error is a vector quantity and it has the following two components. a) Line-of-sight component. This is the error component along the line-of-sight from the user to the satellite. b) Orthogonal component. This is the error component orthogonal to the line-of-sight from the user to the satellite. The orthogonal component will not affect the user navigation solution for calculating the ranging error. The line-of-sight component error is variant because the line-of-sight direction from a user to a given satellite varies depending on the user location. As a consequence the ranging error to satellite varies as a function of user location [11]. As explained earlier, in DGPS, the reference station transmits the ranging errors and range rate errors as corrections to the users. If the variation between the reference station and user is significantly large, it

could cause uncertainties in the extrapolation of errors in the DGPS system. But in the suggested scheme, since the true position of the satellite is transmitted, the user will be able to calculate the ranging errors. Hence the scheme is independent of user location.

As explained in chapter 2 , for real-time DGPS two correction methods are in use. In the first method, the reference receiver at a known location calculates its position using the same set of satellites as the user receiver. The difference of the known position from the calculated one yields position corrections. These values are then transmitted to the user receiver and user receiver thus applies these corrections to obtain an improved position. The second method is based on pseudorange corrections which are derived from the difference between calculated ranges and observed code ranges at reference station. Using this method, the observed code ranges at the user receiver can be corrected by applying fixed reference station corrections.

With the simulated data we have determined the accuracies of the two methods. We have taken four user receivers 5^o apart from the reference station on both sides. We calculated pseudorange error and position error at the reference station. Using these as corrections we calculated the posistions of the users. We introduced random errror into the system and obtained the user position accuracy.

3.4 Generation of data for simulation

We have selected satellite-earth geometry in earth-centred, inertial frame. All satellite orbits are elliptical with the earth's centre at one of the foci. In an inertial co-ordinate frame, the earth's orientation is described in terms of its rotation axis, Z, through the North Pole, its equatorial plane perpendicular to the rotation axis and passing through the earth's centre, its X-axis directed along the 'first point of Aries' (sign of the Ram) and a Y-axis at right angles to the X-axis. The plane of a satellite's orbit cuts the equatorial plane at the point N (refer fig 3.1) making an angle Ω with the X-axis. This angle is known as the 'right ascension of the ascending node' (satellite travelling North-bound). The orbital plane is inclined with respect to the equatorial plane, the angle being known as the inclination i. The point of closest approach to the earth's centre, P, is called the perigee of the orbit and defines ω, the argument of the perigee. At any time, the satellites position, S, within the orbital plane defines and angle NOS. This angle is called

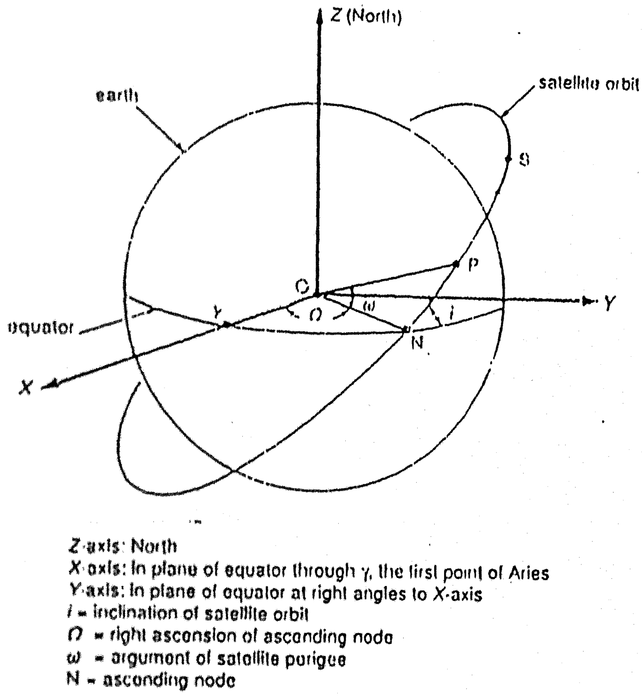


Figure 3.1: Satellite-earth geometry in earth-centred inertial frame

the argument of latitude[2] when referring the satellite's position relative to that of other satellites within the system at a given reference time.

The selected Navstar GPS orbital parameters are given in table 3.1

The Keplerian orbit anomalies, i.e., Mean anomaly M, Eccentric anomaly E, True anomaly θ, are given by

$$\begin{aligned} M &= n(t - T_0) \\ E &= M + esinE \\ \theta &= 2arctan\left[\sqrt{\frac{1+e}{1-e}}tan\frac{E}{2}\right] \end{aligned}$$

where n is mean angular satellite velocity, e is the eccentricity of the orbit, and T_0 is epoch of perigee passage. We have chosen eccentricity e as 0.001, since the GPS satellite orbits are nearly circular . Since the eccentricity is smaller than 0.1, we have used Eccentric anomaly E as

Table 3.1: Selected Navstar orbital parameters

Parameter	Navstar
Inclination, degrees	55.0(block II) 63.0(block I)
Period, minutes	717.97
Semi-major axis, km	26560
Orbital plane separation, degrees	120
Longitude drift per orbit, degrees	180
Ground track repeat, orbits	2
Ground track repeat, days	1
Drift per day, minutes	-4.06
Synchronism period, orbits	16

$$E = M + (e - \frac{e^3}{8})sinM + \frac{1}{2}e^2 sin2M + \frac{3}{8}e^3 sin3M$$

If r,θ are taken as the polar coordinates of the satellite orbit, then r = a(1-ecosE). The coordinates of satellite in earth-centred inertial frame at any given instant of time is given by

$$\begin{aligned}
X_s &= rcos(\theta + \omega)cos\Omega - rsin(\theta + \omega)cos(i)sin\Omega \\
Y_s &= rsin(\theta + \omega)cos(i)cos\Omega + rcos(\theta + \omega)sin\Omega \\
Z_s &= rsin(\theta + \omega)sin(i)
\end{aligned}$$

The argument of latitude and right ascension of the ascending node of GPS satellites are as shown in the fig 3.2 [2].

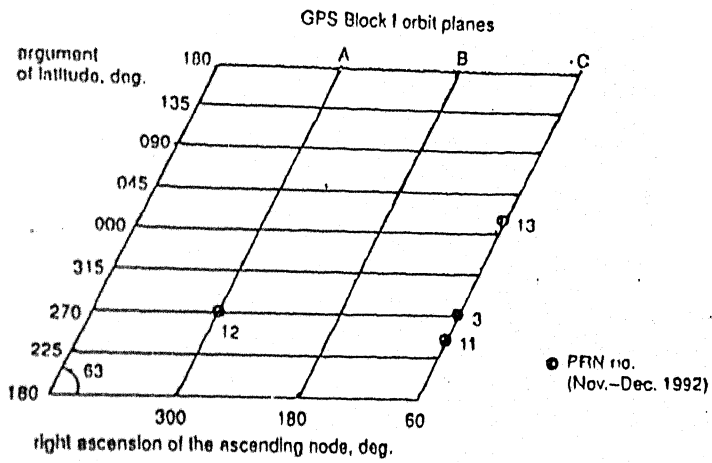
The coordinates of ground reference receiver for a given latitude La, and longitude Lo are given by

$$\begin{aligned}
X_r &= Rcos(La)cos(Lo)cos(w_e t) - Rcos(La)sin(Lo)sin(w_e t) \\
Y_r &= Rcos(La)sin(Lo)cos(w_e t) + Rcos(La)cos(Lo)sin(w_e t) \\
Z_r &= Rsin(La)
\end{aligned}$$

where R is the radius of earth and w_e is the angular velocity of earth. We have used the Greenwich sideral time as on 00:00:00 hours Universal Time on January1,1996. The satellites whose elevation angle is more than 15^o from a station is considered to be visible from that station. By using these data we are able to find out visible satellites from a particular ground reference station and their positions at any instant of time.

3.5 Random error analysis

As explained in chapter 2 (Differential GPS), the pseudorange error consist of bias errors, slowly varying errors and random errors. The source of these errors can be grouped into three. a) satellite based b) propagation based c) the receiver based. The satellite



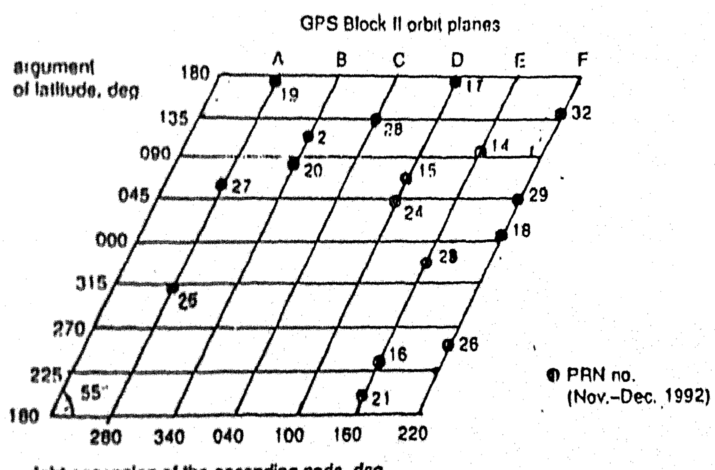


Figure 3.2: GPS satellite orbits in BlockI and BlockII

based errors consist of SA errors, ephemeris and satellite clock errors and there is a random component in the satellite clock error . There are random component errors in troposheric group delay, multipath errors and user receiver. These random errors can not be removed by DGPS. But these can be smoothed by navigation filter and reduced by a factor of 2. In order to test the robustness of the suggested scheme, we introduced random errors in the system. We introduced ±2m in the satellite based errors, ±6m in receiver based errors and ±2m in satellite/receiver combination. It assumes that the random errors are having zero mean and equal variance. This is only approximately true. This can be expressed in the form of a linear equation A x = F, where A is the coefficient matrix, x is the random error in the satellite position vector and F is the random error in the measurement of time vector. The expected value E[x]= A^{-1} E[F]. Then the correlation matrix is given by $E[xx^T]$ =$E[BFF^TB^T]$ where B= A^{-1}. The correlation matrix of random error in the satellite position is given by K_{xx} = B$K_{FF}B^T$, whereK_{FF} is the correlation matrix of random error in the measurement of time.

Chapter 4

Results and discussion

4.1 Determination of SA errors

As explained in chapter 3, for evaluation of SA errors we utilised multi-site reference stations. We have chosen 8 reference stations all over India in such a manner that they cover entire Indian sub-continent. The longitude and latitude of the reference stations are given in the table 4.1.

In order to calculate the SA errors we require minimum of four satellites which are simultaneously visible from all the ground stations. From the available 21 satellites data, at an epoch t=5, we could get 6 satellites (with elevation angle more than 15^o) simultaneouly visible from 6 reference stations. For example fig 4.1 shows the number of satellites visible (zero elevation) at t=15 from Kanpur reference station.

With this we formed 36 equations with 30 unknowns (i.e., 6 Δx, 6 Δy, 6 Δz, the satellite position errors due to SA, 6 δ_{sA} the satellite clock errors due to SA and 6 δ_r the receiver clock errors). We introduced satellite position errors between $\pm$10 km randomly (the satellite position errors are the order of some meters (100-150 m) [3]), the satellite clock error between .1 and .25 μsec and recevier clock error between .2 and .8 μsec. Since the linear equations used are approximate, we used iteration procedure. In the first iteration the accuracy obtained was of the order 10^{-2}km for satellite position errors and clock error accuracy of $10^{-3}\mu$sec. During second iteration the accuracy improved to

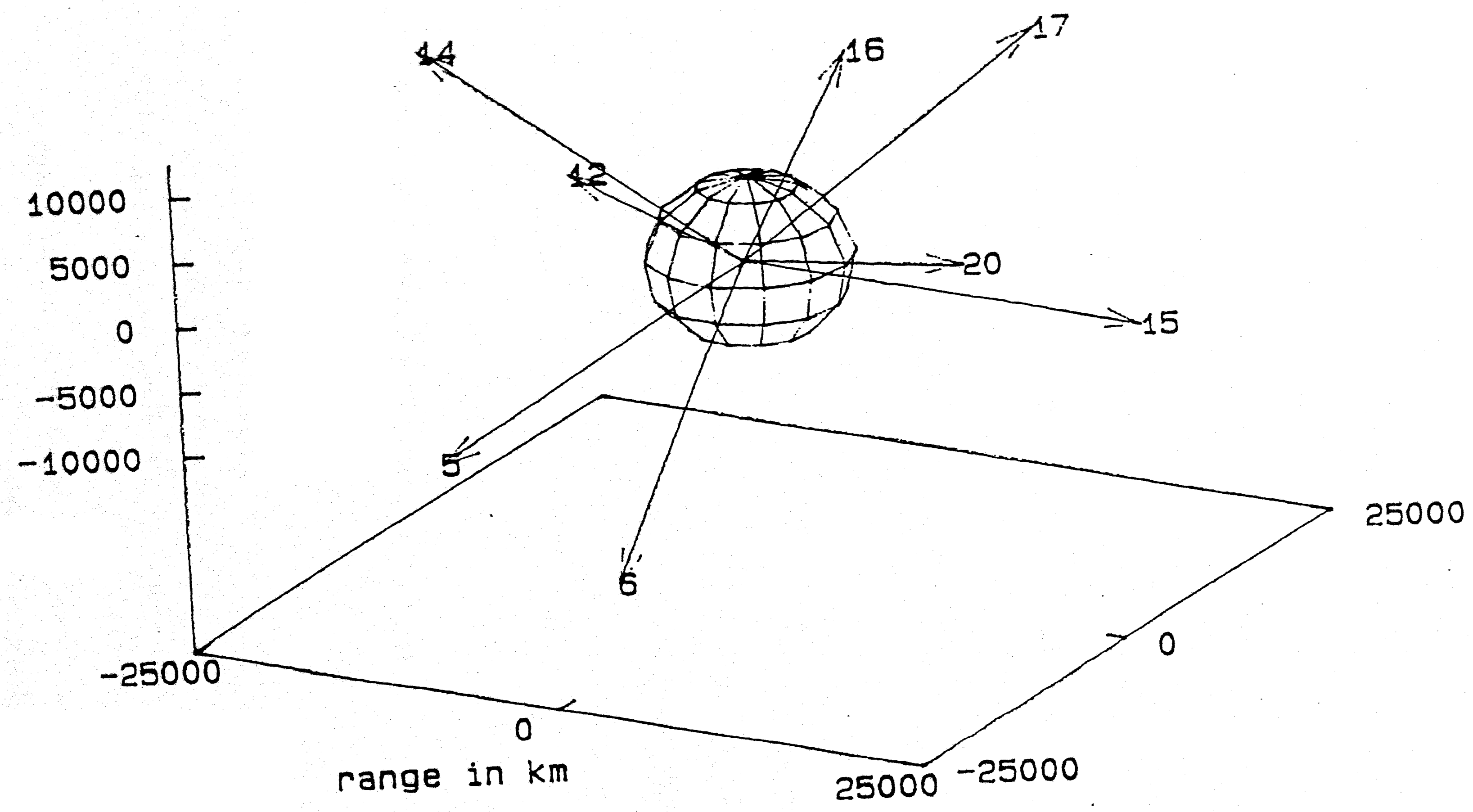


Figure 4.1: Satellites visible from Kanpur at epoch t=15

Table 4.1: Longitude and Latitude of reference stations

Place	longitude(deg)	latitude(deg)
Kanpur	80.3	26.5
Trivandrum	76.9	8.59
Srinagar	74.68	34.37
Rajkot(Gujarat)	70.83	22.35
Dagori(Assam)	95.57	27.67
Padwa(Orissa)	82.72	18.54
Kalayat(Rajastan)	73.00	27.85
Alnavar(Karnataka)	74.78	15.44

10^{-8}km and $10^{-4}\mu$sec.

The same procedure was repeated at other five epochs. The results are of same order of accuracy. We experimented this procedure with the ground stations within ±.1deg of longitude and latitude (base line distance), ±.5deg, ±1deg and ±15deg. The result was that satellite position errors were more when the ground stations are closer. This is given in the fig 4.2

Since there may be rapidly varying components of random errors in troposheric delay, multipath error and in the receiver, there are random errors in the measurement of time (Δ t). We introduced 10m of random error (satellite dependent errors 2 m,receiver depenedent errors 6 m , satellite/receiver combined 2 m)to the right hand side of the equation $Ax = b$. The result was very high inaccuracies (of the order of 10 km) in the satellite positions. This shows that the relevant equations are highly sensitive to random errors appearing in the measurement of pseudoranges which are propagated with very high amplification to satellite errors and only if random errors are reduced by filtering to a great extent, the suggested scheme for determining the true position of the satellite will be successful. An error analysis for the determination of the covariance matrix of errors in estimates of

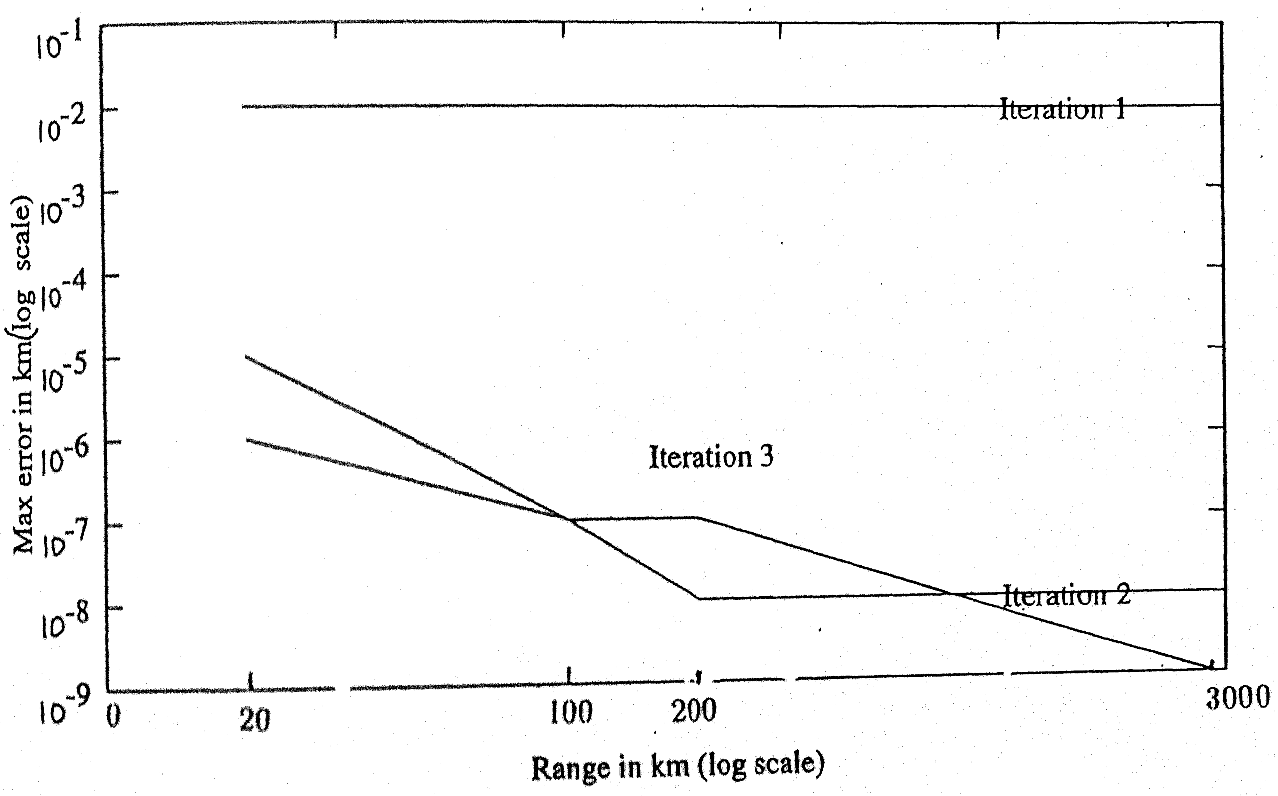


Figure 4.2: Effect of baseline distance on SA error determination

satellite position corresponding to assumed covariance matrix of errors in measurement was made. The results shows that the measurement errors propagates with unacceptable levels of amplification ('18) as the positions of satellites are evaluated by the present method. This is attributable to the small angles (8 ° max) subtended at satellite positions by the cluster of chosen ground station within the country whose maximum distance covers 3000km.

4.2 Estimation of user receiver position

The aim of GPS system is to find the position of the user receiver. The linearised equation for point positioning is given in eqn 3.3. We introduced an error of 100 km in xyz coordinates of the user receiver position as initial estimated range and receiver clock error as .2μsec. Using the approximated linear equations the position of the user was determined. Iteration method was used to improve the accuracy. We simulated SA errors at the satellite randomly over the range of $\pm$ 100 m and measurement errors as 10 m (satellite dependent ± 2 m, receiver dependent ± 6 m and satellite/receiver combined ± 2 m) randomly. Fig 4.3 shows maximum errors (among xyz coordinates of the user receiver) at different combinations of SA error and random error at different user positions within 5^o latitude and longitude.

With SA off (we assumed that true position of the satellites were known) and random error of the accuracy obtained was of the order 10^{-11} km. When SA off and RE (random error) on, the maximum error was the order of 10^{-3} km. Then SA error was introduced and with RE off, the maximum error obtained was 160 m. When SA was on and with RE on, in the extreme case the maximum error obtained was 181 m. This was after three iterations when the solutions converged.

The random error was increased by ten times i.e., to 100m, the results are shown in fig 4.4. It can be seen that when random errors were increased, the accuracy obtained was 250 m.

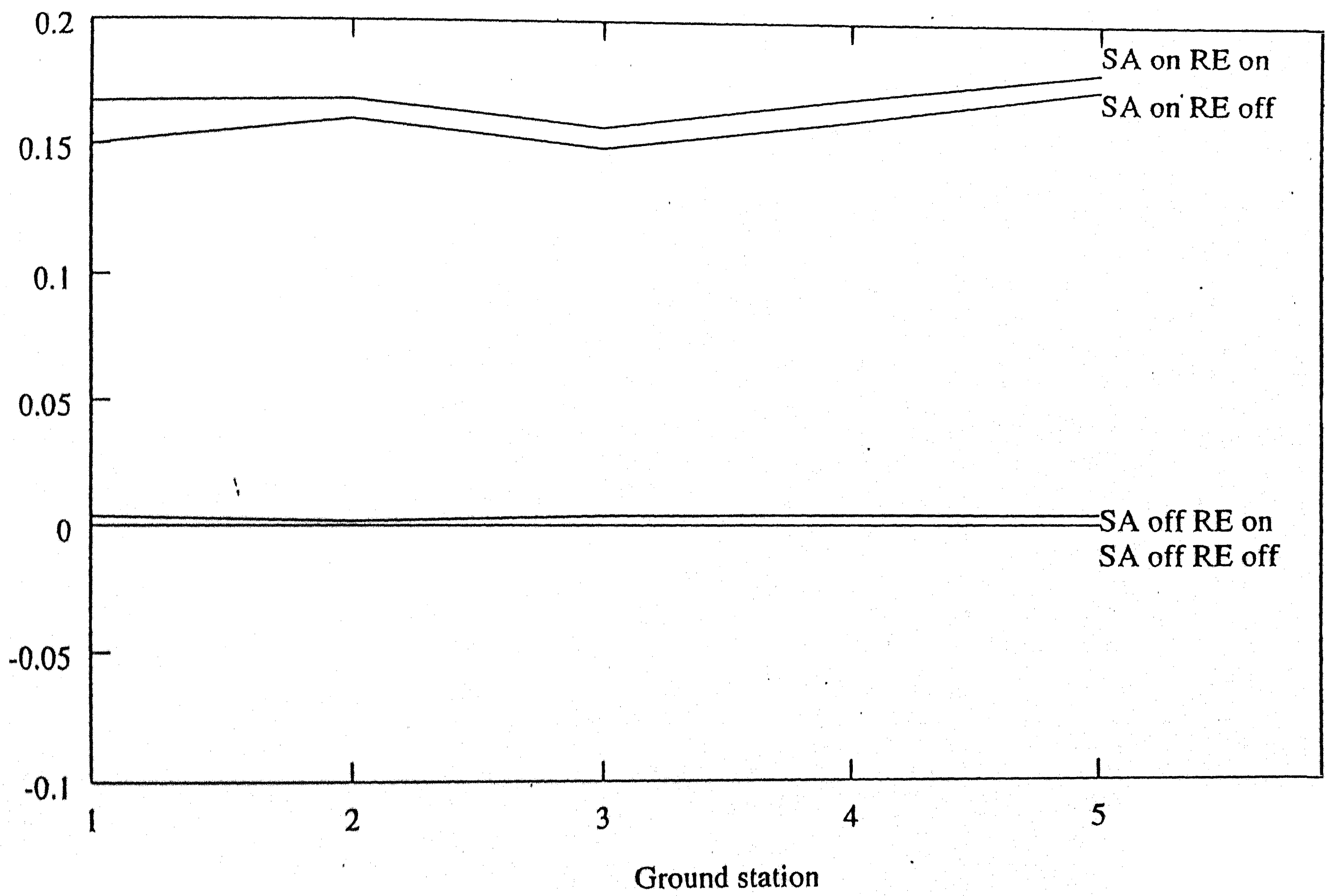


Figure 4.3: Effect of SA(100 m) and random (10 m) errors on positional errors

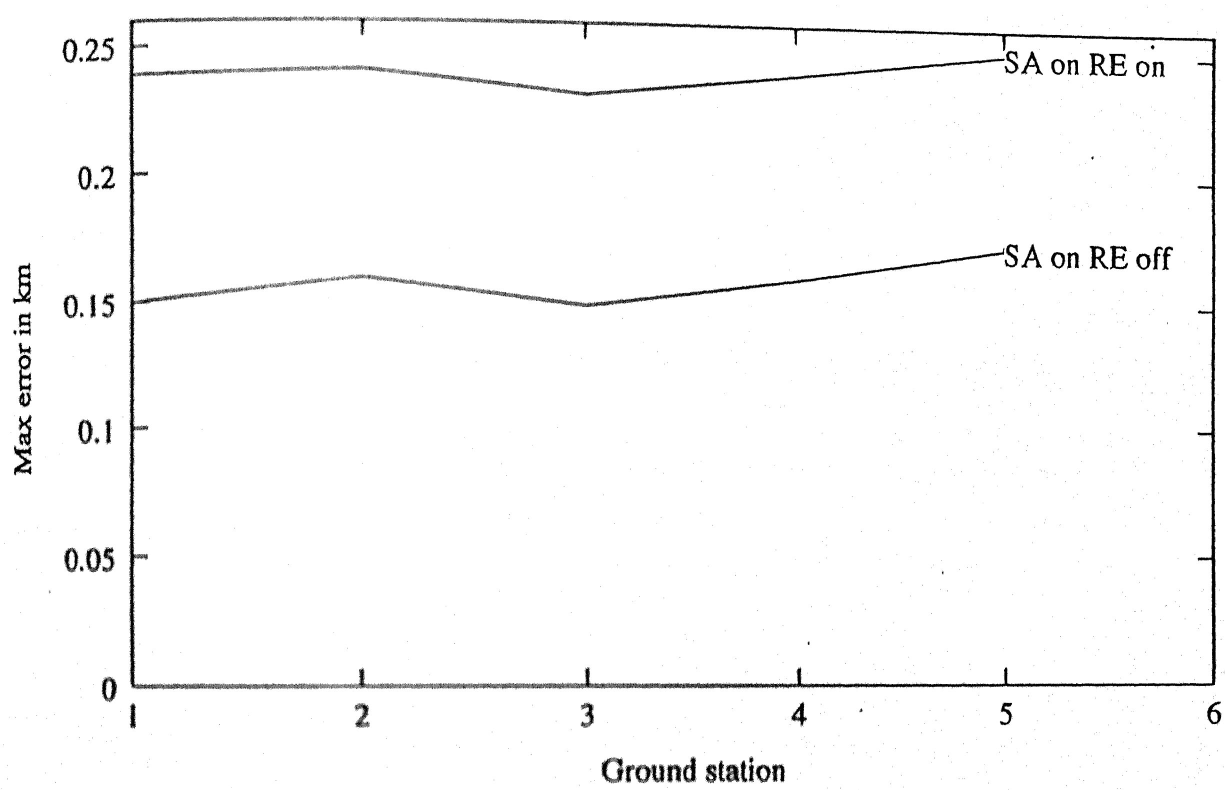


Figure 4.4: Effect of SA(100 m) and RE(100 m) on positional errors

4.3 Differential operation of GPS

In the differential operational mode of GPS (DGPS), we have estimated the errors in pseudo range as well as in position errors with respect to reference ground station. Then we have transferred these errors to the user receivers for updating their positions. We have taken four user receivers at 5^o (longitude and latitude) variation from the reference station. A random error of 10 m and an SA error of ± 100 m (randomly) were introduced. Then the pseudorange error (difference between the true range and estimated range to the satellite from the reference station) was estimated. The same set of satellites which were used for the determination of the pseudorange error of reference ground station were used to find out the position of the user receivers. While evaluating the position of the user a fixed correction corresponding to the corrections (pseudoranges, or positions) required at the reference station was applied. The errors obtained at diferent user receivers are as given in fig 4.5.

The maximum error obtained in pseudorange error transfer method was 35 m and in positional error transfer method was 15 m.

We increased the random error to 100 m and obtained the results. It is given in the fig 4.6.

It can be seen that the errors were increased in both the methods to a level of 290 m in pseudo range error transfer and 70 m in positional error transfer methods.

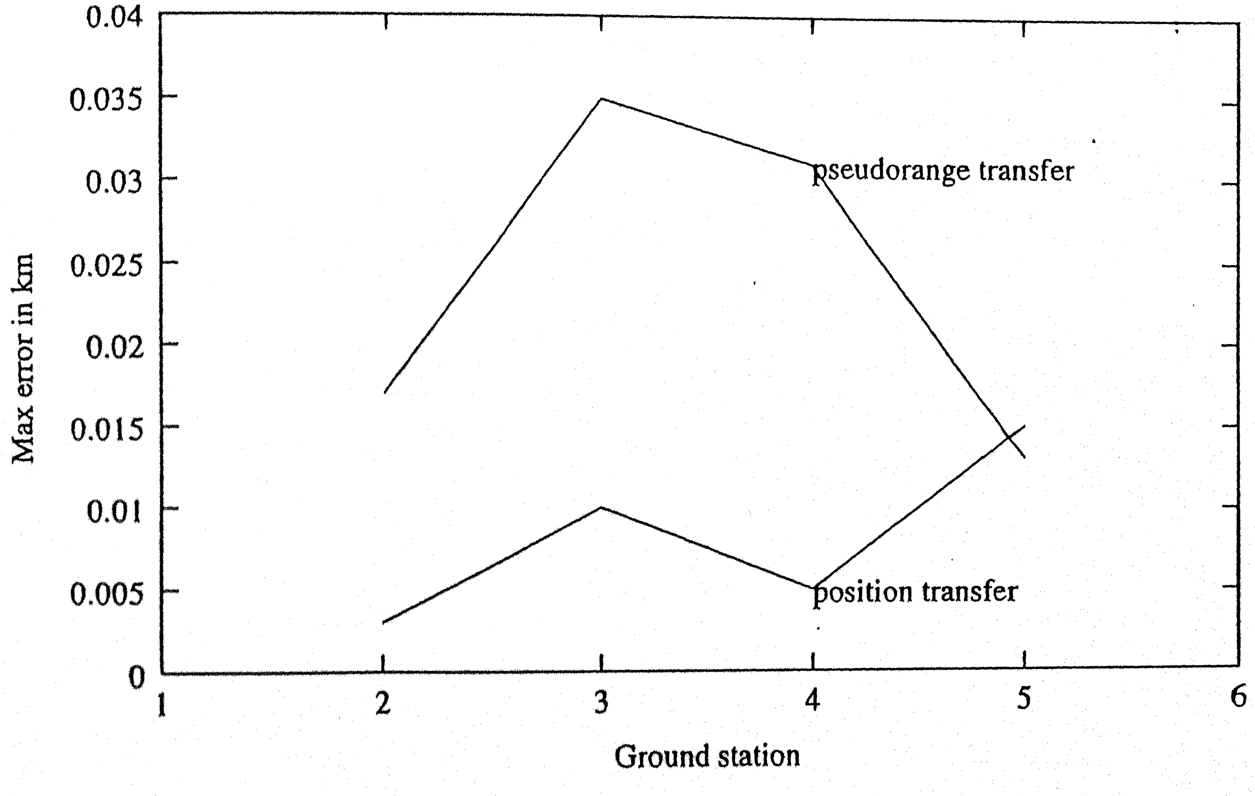


Figure 4.5: Positional errors in DGPS operation (RE 10 m and SA 100 m)

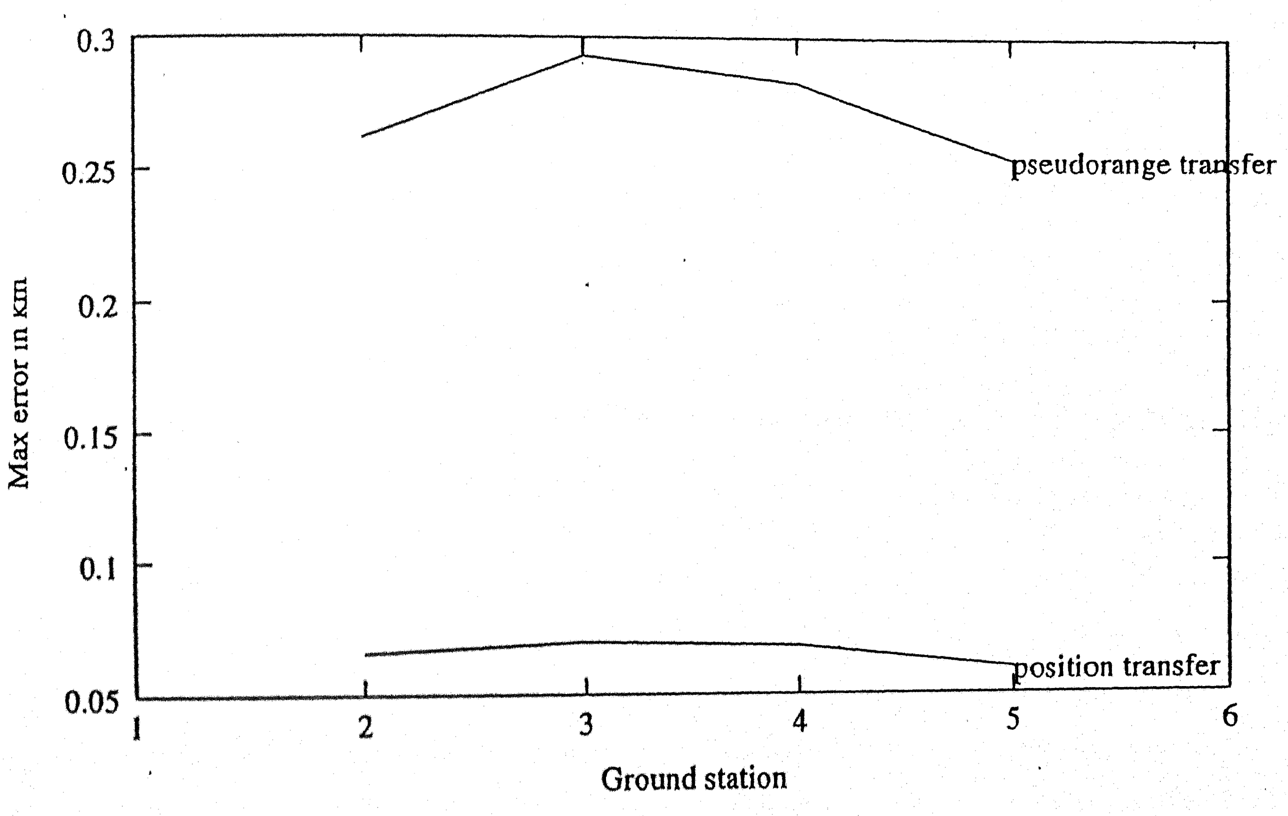


Figure 4.6: Errors in DGPS mode(RE 100 m and SA 100 m)

Chapter 5

Summary and Conclusions

An attempt has been made in the present study to directly determine, by code pseudorange measurements, the Selective Availability (SA) errors of satellites of NAVSTAR system with the aid of eight ground monitoring stations spread across India. This was done with a view that once the SA errors are determined with fair accuracy and broadcast, user receivers independent of their location, can make necessary corrections to the estimates of their respective positions.

The proposed method for determination of SA errors by linearisation of geometric equations was successful when random measurement errors were ignored. It was very robust for large values of SA errors, estimating the SA errors to a higher degree of accuracy ($10^{-}9$ km). However when random measurement errors were simulated, the method failed to estimate SA errors which were actually amplified during the estimation. This may be attributed due to the limited spread (max 3000km) of reference ground stations. The result may improve if reference stations are spread over a larger area, covering more countries in the hemisphere containing the Indian subcontinent. However more refined procedures of filtering , smoothing and estimation, followed in satellite orbit determination practices may have to be used.

In the second part of this study, the effects of SA and random errors in range measurements, on point positioning of receiver were investigated by simulation of these errors, for one epoch of the NAVSTAR GPS and 6 ground stations located $\pm 5^o$ around Kanpur. It was observed that the variation of SA errors over a region of 500-600 km is small.

Hence if the position error due to SA is determined at one ground station, then it may be used at all stations over the region by direct transfer of the necessary correction without sacrificing much accuracy.

In the last part, an evaluation of two methods of correction viz. position correction transfer and pseudorange correction transfer, for SA errors by differential operation of GPS is made. It was found that the position correction transfer method yeilded better accuracy than pseudorange correction transfer.

5.1 Recommendations for future work

Extensive simulation studies,using more number of satellie/receiver configurations and at more epochs will better validate the results of the study. A study of single, double and triple difference methods for compensation of SA errors may also be made.

The code pseudorange measurements considered in the present study may be supplemented by measurements of the associated carrier phases which will improve the accuracy.

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